SHENTON COLLEGE

Year 11 Mathematics Methods
AEMAM Term 1 2023

Test 1 Counting and Probability

Calculator Free ***Formula Sheet Allowed***

Student Name: MARKING KEY

Teacher (circle): Cheshire Coveney McKenna McRae Moore Munro

Time Allowed: 25 minutes

Calculator Free:	/27

*Attempt **all** questions.*
*All necessary working and reasoning must be shown for **full marks**.*
Marks may not be awarded for untidy or poorly arranged work.

Question 1. **[3 marks]**

(a) Evaluate:

(i) $\frac{11!}{9!}$

$= 11 \times 10$

$= 110$

① correct value

(ii) ${}^{7}C_{3}$

$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1}$

$= 35$

① correct value

(iii) $\binom{n}{k} \div \binom{n}{n-k}$

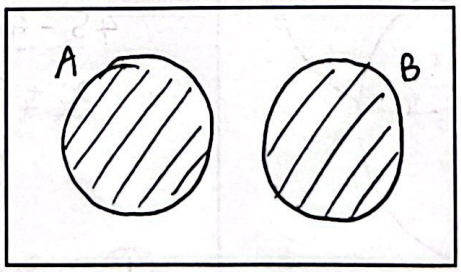
$= 1$

① correct value

Question 2. **[2, 1 = 3 marks]**

(a) Two events A and B are such that $A \cap B = \emptyset$.

(i) Draw a Venn diagram to show the relationship between A and B and shade the region representing $A \cup B$.



① Draws & labels circles with no overlap

① shades AUB

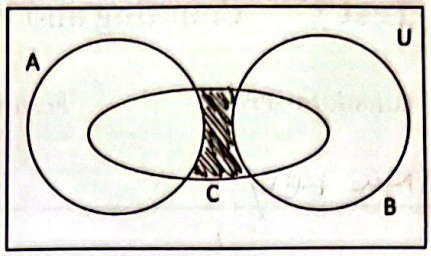
(ii) Describe the relationship between the events A and B.

mutually exclusive

① states ME

Question 3. **[1, 1, 3 = 5 marks]**

(a) Shade the region of the Venn diagram below that represents $C \cap (\overline{A \cup B})$.



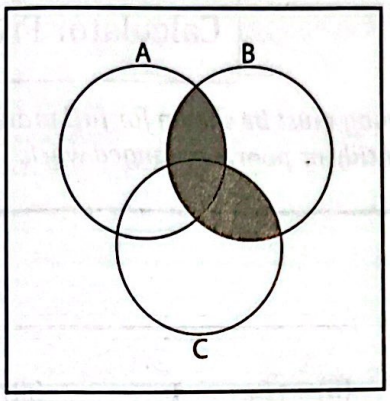
① correct region shaded

(b) Use set notation to describe the shaded region in the Venn diagram below.

$(A \cup C) \cap B$

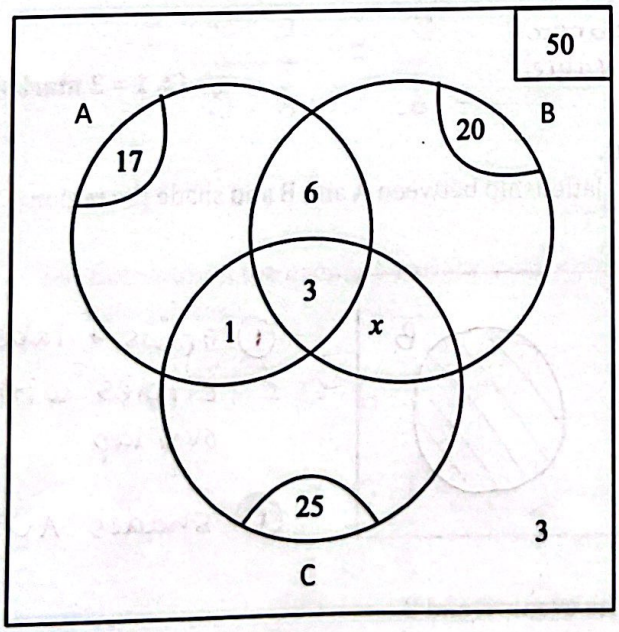
or

$A \cap B \cup B \cap C$



① correct expression

(c) Determine the value of x in the Venn diagram below



$A \cap (B \cup C) = 17 - (6 + 3 + 1) = 7$

$B \cup C = 50 - (7 + 3) = 40$

$B \cap C = (20 + 25) - 40$

$= 5$

$\therefore \quad x + 3 = 5$

$x = 2$

① Determines missing value in A

① Creates expression for $B \cup C$ or $A \cup B \cup C$

① solves for x

2

Question 4. **[2, 1, 3, 2 = 8 marks]**

Events A and B are such that $P(A) = 0.6$, $P(B) = 0.5$ and $P(A' \cap B) = 0.4$

	A	A'	TOTAL
B	0.1	0.4	0.5
B'	0.5	0	0.5
TOTAL	0.6	0.4	1

① inputs information correctly

① determines missing values.

(a) Complete a two-way table based on the information above.

(b) Explain why the events A' and B' must be mutually exclusive.

$P(A' \cap B') = 0$ ∴ events cannot both happen

① refers to probability of intersection = 0

(c) Determine

(i) $P(\bar{A} \cup \bar{B})$

$0.4 + 0.5 = 0.9$ ① correct value

(ii) $P((\overline{A \cap B})|A)$

$= \frac{0.5}{0.6} = \frac{5}{6}$

① determine $P((\overline{A \cap B}|A))$

① correct simplified probability

(d) Determine if the events A and B are independent, supporting your answer with appropriate calculations.

$P(A) = 0.6$

$P(A|B) = \frac{0.1}{0.5} = 0.2$

$0.2 \neq 0.6$

∴ events are not independent

① Substitutes values into relevant formula

① States not independent with reference to result

Question 5. **[1, 1, 1 = 3 marks]**

Let U denote the group of all students who visited Europe over the school holidays. If:

- I represents the set of students that visited Italy,
- E represents the set of students that visited England
- G represents the set of students that visited Germany

describe in words, which students are represented by the following sets.

a) $I \cup E \cup G$

Those that visited one or more of Italy, England and Germany

b) $\overline{I \cap E \cap G}$

Those that did not visit all three of Italy, England Germany

c) $(E \cup G) \cap I'$

Those that visited England and/or Germany but did not visit Italy.

Valid Description ① each

Question 6. **[1, 2, 2 = 5 marks]**

(a) State the five terms in row 4 of Pascal's triangle.

$$\binom{4}{0}\binom{4}{1}\binom{4}{2}\binom{4}{3}\binom{4}{4} = 1 \quad 4 \quad 6 \quad 4 \quad 1$$

① states correct values or notation

(b) Expand $(a-b)^4$

$$a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

① correct coefficients (including signs)

① correct powers

(c) Determine the coefficient of x^4 in the expansion of $\left(4 - \frac{x}{2}\right)^7$

$$\binom{7}{3}(4)^3\left(-\frac{x}{2}\right)^4 = \frac{(35)(4)^3 x^4}{2^4}$$

$$= 140\,x^4$$

Coefficient = 140

① correct expression for x^4 term or coefficient

① evaluates expression

End of Calculator-Free Section

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Question 7. **[1, 1, 2, 1 = 5 marks]**

Shen-Car insurance company collected data on two different driver age groups who had made an insurance claim over a 1-year period. Some of the data is shown in the table below.

AGE OF DRIVER:	17-25	26-40	TOTAL
CLAIM	220	150	370
NO CLAIM	180	450	630
TOTAL	400	600	1000

(a) Complete the table ① correct

(b) How many drivers did not make a claim?

630 ① correct from table

(c) What is the probability a randomly selected driver:

(i) Is aged 26-40 and didn't make a claim.

$\frac{450}{1000} = \frac{9}{20}$ ① correct from table

(ii) Is aged 17-25 or made a claim.

$\frac{550}{1000} = \frac{11}{20}$ ① correct from table

(d) Determine the proportion of drivers aged 17-25 who made a claim during the year.

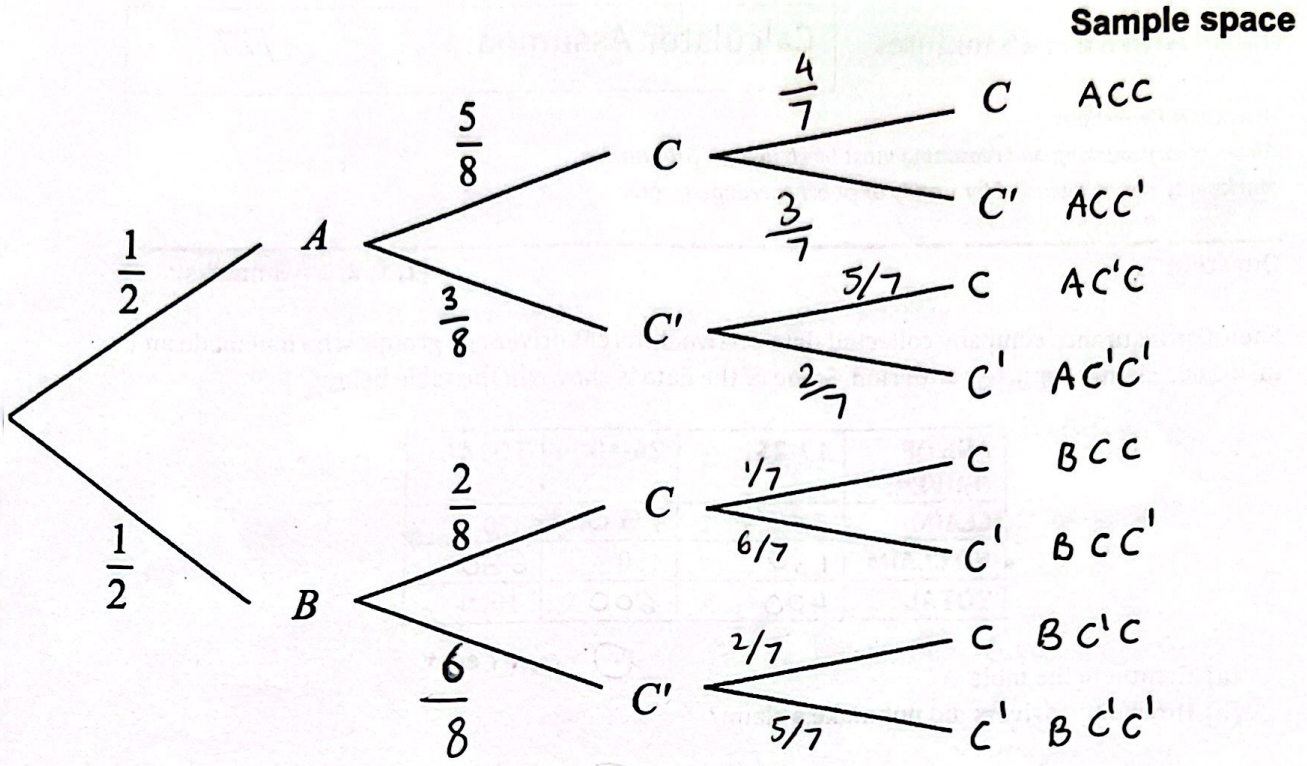
$\frac{220}{400} = \frac{11}{20}$ ① correct from table

Question 8. **[3, 2, 2 = 7 marks]**

Two prize boxes each contain 8 envelopes. There are five envelopes in Box A that each contain $100 cash prizes. There are two envelopes in Box B that each contain $100 cash prizes. All other envelopes are empty.

Connor selects one box at random and then draws two envelopes, randomly, without replacement.

(a) Complete the weighted tree diagram and sample space for this scenario, where C represents selecting an envelope containing cash.



① Determines complementary probability ① Recognises non-replacement ① Lists appropriate possible outcomes

(b) Calculate the probability Connor wins $200.

$P(ACC) + P(BCC)$ ① Lists both possible outcomes

$= \frac{1}{2} \times \frac{5}{8} \times \frac{4}{7} + \frac{1}{2} \times \frac{2}{8} \times \frac{1}{7} = \frac{22}{112}$ ① Evaluates probability

(c) If Connor won $100 from his first envelope, what is the probability he chose box A?

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{5}{16} \div \left(\frac{5}{16} + \frac{2}{16}\right)$$

① uses conditional formula in context $= \frac{5}{7}$ ① evaluates probability

Question 9. **[1, 1, 1, 3, 2, 2 = 10 marks]**

A committee of 7 people is being selected from senior members at a local sports club consisting of 7 frisbee players, 8 roller derby players and 5 lacrosse players.

(a) In how many ways can the 7 people be selected if:

(i) There are no restrictions

$20C_7 = 77520$ ① correct expression

(ii) Only roller derby players are selected

$8C_7 = 8$ ① correct expression

(iii) No roller derby players are selected

$12C_7 = 792$ ① correct expression

(iv) At least 2 representatives from each sport are selected.

3F 2R 2L : $7C_3 \times 8C_2 \times 5C_2 = 9800$ +

3R 2F 2L : $8C_3 \times 7C_2 \times 5C_2 = 11760$

3L 2R 2F : $5C_3 \times 7C_2 \times 8C_2 = 5880$

① Demonstrates use of multiplication principle

① Demonstrates appropriate method

At least 2 from each : $9800 + 11760 + 5880$

$= 27440$

① Evaluates additive or complementary expression

(b) Two roller derby players, Ruby Bruiseday and Drew Bash 'emore find out that exactly 4 roller derby players have been selected for the committee. Calculate the probability:

(i) Ruby has been selected

4 spots : $8C_4$ combinations $= 70$

∴ no. of committees containing Ruby $= 7C_3 = 35$

∴ $P(\text{Ruby}) = 1/2$

① Determines # of committees with Ruby

① Provides correct probability

(ii) Only one of them has been selected

$$\frac{\text{Ruby/Drew} \times \text{Remaining Spots}}{\text{Total \# of 4 derby committees}}$$

① Uses multiplication principle correctly

$$= \frac{2C_1 \times 6C_3}{8C_4} = \frac{4}{7}$$

① Answers as probability

Question 10. **[5 marks: 1, 2, 2]**

Classified documents at the nation's leading intelligence agency *SP135* are protected by three different anti-hack programs (referred to as A, B, and C for security reasons). Each program has the following chance of being hacked:

$$P(A) = 0.3 \quad P(B) = 0.15 \quad P(C) = 0.1$$

One program being hacked does not affect the probability of another program being hacked.

Determine the probability that during a cyber-attack:

(a) All programs will be hacked.

$P(A \cap B \cap C) = 0.3 \times 0.15 \times 0.1$

$= 0.0045$ ① evaluates probability

(b) At least one of the programs will be hacked.

$P(A' \cap B' \cap C') = 0.7 \times 0.85 \times 0.9$

$= 0.5355$ ① Demonstrates appropriate method

$P(\text{At least 1}) = 1 - P(\text{none}) = 1 - 0.5355 = 0.4645$

① Evaluates probability

(c) Exactly two of the programs will be hacked.

$P(ABC' \cup AB'C \cup A'BC)$ ① identifies possible combinations

$= 0.3 \times 0.15 \times 0.9 + 0.3 \times 0.85 \times 0.1 + 0.7 \times 0.15 \times 0.1$

$= 0.0405 + 0.0255 + 0.0105$

$= 0.0765$ ① Evaluates probability

End of Calculator Assumed Section